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# SUPERSYMMETRIC ELECTROWEAK RENORMALIZATION OF THE $Z$ -WIDTH IN THE MSSM (I)

David GARCIA <sup>1</sup>, Ricardo A. JIMÉNEZ <sup>2</sup>, Joan SOLÀ <sup>3</sup>

Grup de Física Teòrica

and

Institut de Física d'Altes Energies

Universitat Autònoma de Barcelona

08193 Bellaterra (Barcelona), Catalonia, Spain

## ABSTRACT

Within the framework of the MSSM, we compute the complete set of electroweak one-loop supersymmetric quantum effects on the width  $\Gamma_Z$  of the  $Z$ -boson in the on-shell renormalization scheme. Numerical analyses of the corrections to the various partial widths into leptons and quarks are presented. On general grounds, the average size of the electroweak SUSY corrections to  $\Gamma_Z$  may well saturate the level of the present theoretical uncertainties, even if considering the full supersymmetric spectrum lying in the neighbourhood of the unaccessible LEP 200 range. Remarkably enough, for the present values of the top quark mass, the electroweak SUSY effects could be, globally, very close or even bigger than the electroweak SM corrections, but opposite in sign. Therefore, in the absence of theoretical errors, there are large regions of parameter space where one could find that, effectively, the electroweak SM corrections are “missing”, or even having the “wrong” sign. This should be helpful in discriminating between the SM and the MSSM. However, an accurate prediction of the electroweak quantum effects on  $\Gamma_Z$  will only be possible, if  $\Delta r$  and  $\alpha_s$  are pinned down in the future with enough precision.

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<sup>1</sup>Internet address:GARCIA@IFAE.ES

<sup>2</sup>Internet address:ABRAHAM@IFAE.ES

<sup>3</sup>Internet addresses:IFTESOL@CC.UAB.ES and SOLA@IFAE.ES

An outstanding laboratory to test possible manifestations of Supersymmetry (SUSY) and particularly of the Minimal Supersymmetric Standard Model (MSSM) [1] is LEP, either at  $Z$ -pole energies or in the near future also from on-shell  $W$ -physics. If no direct production of supersymmetric particles (“sparticles”) is achieved neither at LEP 100 nor at LEP 200, still some indirect manifestations could be discovered from quantum effects [2]. In fact, radiative corrections to conventional physical processes [3] are a powerful tool to search for mass scales within and beyond the Standard Model (SM), and they offer us the opportunity to peep at sectors of the theory that are not (yet) directly observable. In this respect it is useful to remember that at LEP 200 the  $W$ -mass will be measured with a remarkable precision of  $\delta M_W = \pm 28\text{ (stat.)} \pm 24\text{ (syst.) MeV}$  [4]. Recent analyses [5, 6] have shown that a measurement of the  $W$ -mass with that precision, or even a factor of two worse, would enable us to hint at virtual SUSY effects even if the full supersymmetric spectrum lies in the vicinity of the unaccessible LEP 200 range ( $\gtrsim 100\text{ GeV}$ ).

Similarly, on-shell  $Z$ -physics is also sensitive to quantum effects from sparticles. The mass and width of the  $Z$ -boson are experimentally known with particularly good accuracy [7]

$$M_Z^{\text{exp}} = 91.1895 \pm 0.0044\text{ GeV} \quad (1)$$

and

$$\Gamma_Z^{\text{exp}} = 2.4969 \pm 0.0038\text{ GeV}. \quad (2)$$

One expects that the  $Z$ -width will eventually be measured within only  $|\delta\Gamma_Z^{\text{exp}}| \lesssim 2\text{ MeV}$  [7]. In the on-shell scheme [8] the  $Z$ -mass enters as an experimental input, while the  $Z$ -width can be predicted with great accuracy. A detailed calculation of the electroweak one-loop effects on  $\Gamma_Z$  in the SM is given in refs.[9, 10]. The final updated numerical result, including QCD corrections, reads [11]

$$\Gamma_Z^{SM} = 2.4922 \pm 0.0075 \pm 0.0033\text{ GeV}, \quad (3)$$

where the label SM stresses that this theoretical result drops from the strict Standard Model with minimal (single) Higgs sector. The first error corresponds to the variation with the top quark and Higgs mass within the allowed range of  $\delta M_W$  and  $\Delta r$  [11], and the second error is the hadronic uncertainty from  $\alpha_s = 0.123 \pm 0.006$  measured from hadronic event topologies at the  $Z$ -peak. Inclusion of the recently claimed CDF value for the top quark mass ( $m_t = 174 \pm 10^{+13}_{-12}\text{ GeV}$  [12]) leads to [11]

$$\Gamma_Z^{SM}|_{CDF} = 2.4933 \pm 0.0064 \pm 0.0033\text{ GeV}. \quad (4)$$

In spite of how respectably well the SM prediction (3)-(4) matches the experimental result (2), the inherent theoretical and experimental uncertainties still leave room enough

to allocate hypothetical new contributions beyond the SM, such as those from the MSSM. Indeed, in the MSSM we expect a different theoretical prediction for the  $Z$  width,  $\Gamma_Z^{MSSM}$ , which may be conveniently split up into two pieces

$$\Gamma_Z^{MSSM} = \Gamma_Z^{RSM} + \delta\Gamma_Z^{MSSM}, \quad (5)$$

where  $\Gamma_Z^{RSM}$  involves the contribution from a so-called ‘‘Reference SM’’ (RSM) [13]: namely, the Standard Model with the single Higgs mass set equal to the mass of the lightest  $CP$ -even Higgs scalar of the MSSM, whereas  $\delta\Gamma_Z^{MSSM}$  constitutes the total quantum departure of the MSSM prediction with respect to that Reference Standard Model. Besides,  $\delta\Gamma_Z^{MSSM}$  itself splits up naturally into two parts, viz. the extra two-doublet Higgs contribution  $\delta\Gamma_Z^H$ , in which the single Higgs part included in  $\Gamma_Z^{RSM}$  has been subtracted out in order to avoid double-counting, and the SUSY contribution  $\delta\Gamma_Z^{SUSY}$  from the plethora of ‘‘genuine’’ ( $R$ -odd) supersymmetric particles:

$$\delta\Gamma_Z^{MSSM} = \delta\Gamma_Z^H + \delta\Gamma_Z^{SUSY}. \quad (6)$$

Difficult enough, the new effects have to be disentangled from the yield of conventional sectors of the theory which are not yet experimentally determined with enough precision, most conspicuously the top quark mass <sup>4</sup>. Furthermore, as it has been pointed out in other contexts [2], the quantum corrections in the MSSM, quite independently from the values of the various parameters, may mimic those in the SM. Therefore, to effectively discriminate the theoretical predictions of the MSSM from those of the SM is a rather delicate matter that requires to compare the simultaneous predictions on several observables, as for example  $\Gamma(Z \rightarrow f\bar{f})$  and  $M_W$ . However, whereas the full treatment of the SUSY corrections to the  $W$ -mass in the on-shell scheme has already been accomplished in detail by several groups [2, 5, 6], the corresponding corrections to  $\Gamma_Z$  have been partially computed [2, 15] on only some specific decay channels and/or explicitly ignoring the effects from parts of the SUSY spectrum and/or considering only leading effects (e.g. large Yukawa couplings). Particularly interesting by itself is the study of the additional contributions to  $\Gamma_Z$  from two-Higgs-doublet-model extensions of the SM [16]. Although there is some work in the literature for the general unconstrained case and for the supersymmetric case [17, 18], a systematic analysis of the latter incorporating a detailed treatment of the mass relations in the MSSM Higgs sector is lacking. Therefore, in this note we would like to settle down these matters on the basis of an exact one-loop calculation of the electroweak part of  $\delta\Gamma_Z^{MSSM}$  by keeping all effects from gauge and Yukawa couplings and for arbitrary values of the parameters. The calculation of this quantity is

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<sup>4</sup>Potentially significant virtual hints from SUSY have been recently recognized on the physics of top quark decay [14].

indeed a rather complex task. We believe, also an important task. We have faced it in full, with the double purpose of completing previous calculations and at the same time to assess the real possibilities of SUSY to give a hint of existence from the high precision world of  $Z$  physics. To our knowledge, no systematic analysis of the impact of the electroweak genuine SUSY sector of the MSSM on  $\Gamma_Z$ , split up into the various partial widths, has been clearly put in a nutshell anywhere in the literature. Our intention is to numerically demonstrate that, despite of the fact that all SUSY effects must decouple for large enough sparticle masses, we may still expect potentially measurable supersymmetric electroweak contributions (i.e. contributions that could be near the present theoretical errors and well above the planned experimental accuracy  $|\delta\Gamma_Z^{\text{exp}}| \lesssim 2 \text{ MeV}$ ) even for an average SUSY spectrum that surpasses the LEP 200 discovery range. These corrections have to be added to possible SUSY-QCD corrections [19] mediated by gluinos, which are generally smaller and of the same sign. On the whole, this should help to untangle the differences between the SM and the MSSM, especially when combined with the predictions on other observables like  $M_W$  and the asymmetries at the  $Z$ -pole. We divide our presentation into two parts: i) In the present part (Part I) a full account of the “genuine” ( $R$ -odd) SUSY contributions; namely, from sfermions (squarks and sleptons) and “inos” (charginos and neutralinos), is considered in detail. They constitute the complete supersymmetric electroweak radiative shift  $\delta\Gamma_Z^{\text{SUSY}}$  in eq.(6) and we find that, globally, they could provide a source of relatively large loop contributions, in particular if the sparticles are not too heavy. For definiteness, our numerical analysis follows the same pattern of sparticle masses as defined by the so-called Models I and II in Ref.[5], where we analyzed the full contribution to  $\Delta r^{\text{SUSY}}$ . These models are general enough to comprehend both phenomenological as well as more restricted (supergravity inspired [1]) models. On the other hand, in Part II [20], which we present in a separate note following this one, we consider the analysis of  $\delta\Gamma_Z^H$  within the framework of an improved (one-loop corrected [21]) MSSM Higgs sector and compare with  $\delta\Gamma_Z^{\text{SUSY}}$ . The result is especially significant for the  $b\bar{b}$  channel and the associated ratio  $R_b$ , since its experimental value could be in discrepancy with the SM prediction [7]. However, we postpone the explicit presentation of our analysis of the full width (5) of the  $Z$  in the Minimal Supersymmetric Standard Model for a separate and lengthy forthcoming publication where the gory details of the present calculation can be found, together with a more comprehensive exposition of the numerical results [22]. In the meanwhile, our notation for the SUSY formalism follows Ref.[14] and also the early work of Ref.[27].

As stated, our computation of  $\delta\Gamma_Z^{\text{SUSY}}$  is carried out in the on-shell renormalization scheme <sup>5</sup>, where the fine structure constant,  $\alpha \equiv \alpha_{\text{em}}(q^2 = 0)$ , and the phys-

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<sup>5</sup>For a detailed review, see e.g. refs.[3, 23, 24].

ical masses of the gauge bosons, fermions and scalars are the renormalized parameters:  $(\alpha, M_W, M_Z, M_H, m_f, \dots)$ . We will, for brevity sake, refer to it as the  $\alpha$ -scheme:  $(\alpha, M_W, M_Z)$ . In practice, in order to achieve higher accuracy in the theoretical predictions it is convenient to adopt the constrained  $\alpha$ -scheme  $(\alpha, G_F, M_Z)$  in which one substitutes the high precision effective parameter  $G_F$  (Fermi's constant in  $\mu$ -decay) for  $M_W$  by means of the constraint

$$\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{2 M_Z^2 s^2 c^2} \frac{1}{1 - \Delta r^{MSSM}}, \quad (7)$$

where  $c \equiv M_W/M_Z$  and  $s^2 \equiv 1 - c^2$ . In this equation,

$$\Delta r^{MSSM} = \Delta r(\alpha, M_W, M_Z, M_H, m_f, m_{SUSY}, \dots), \quad (8)$$

where  $m_{SUSY}$  is a generic soft SUSY-breaking parameter which sets the characteristic mass scale of the various sparticles.  $\Delta r^{MSSM}$  embodies the resultant finite effect from all possible radiative corrections, universal (U) and non-universal (NU) to  $\mu$ -decay in the MSSM:

$$\Delta r^{MSSM} = \Delta r^U + \Delta r^{NU} = -\frac{\hat{\Sigma}_W(0)}{M_W^2} + \Delta r^{NU}. \quad (9)$$

For our purposes, it will also be useful to split  $\Delta r^{MSSM}$  as follows

$$\Delta r^{MSSM} = \Delta r^{RSM} + \delta(\Delta r)^{MSSM}, \quad (10)$$

with

$$\delta(\Delta r)^{MSSM} = \Delta r^H + \Delta r^{SUSY}, \quad (11)$$

the meaning of the terms on the RHS of eqs.(10)-(11) being fully parallel to those in eqs.(5)-(6). The renormalized self-energy of the  $W$ -boson at zero frequency is given by

$$\hat{\Sigma}_W(0) = \Sigma_W(0) + \delta M_W^2 + M_W^2 \left\{ \frac{\Sigma^\gamma(k^2)}{k^2} \Big|_{k^2=0} - 2 \frac{c}{s} \frac{\Sigma^{\gamma Z}(0)}{M_Z^2} + \frac{c^2}{s^2} \left( \frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2} \right) \right\}. \quad (12)$$

Here  $\Sigma^{\gamma, W, Z, \dots}(k^2)$  are the real parts of the (transverse components of the unrenormalized) gauge boson self-energy functions. Finally, the gauge boson mass counterterms

$$\delta M_W^2 = -\Sigma^W(k^2 = M_W^2), \quad \delta M_Z^2 = -\Sigma^Z(k^2 = M_Z^2) \quad (13)$$

are enforced by the on-shell renormalization conditions. From these equations one usually decomposes [3]

$$\Delta r^U = \Delta\alpha - \frac{c^2}{s^2} \Delta\rho + \Delta r_{rem.}, \quad (14)$$

whose interpretation in terms of the renormalization group (RG) running of  $\alpha$  ( $\Delta\alpha$ ) and the various statical ( $\Delta\rho$ ) and dynamical ( $\Delta r_{rem.}$ ) contributions to the breaking of global

$SU(2)$  symmetries (such as custodial symmetry [25]) is well known in the literature [26]. As for the explicit analytic expressions of the SUSY contributions to the above formulas we use the results of Ref.[27], which we shall not repeat here<sup>6</sup>.

The one-loop partial width of the  $Z$ -boson into a fermion-antifermion pair can be expressed generically in the vector-axial representation, and in the  $G_F$ -parametrization, in terms of two form factors  $\rho_f = 1 + \delta\rho_f$  and  $\kappa_f = 1 + \delta\kappa_f$  as follows:

$$\Gamma(Z \rightarrow f\bar{f}) = N_c^f \frac{G_F M_Z^3}{24\pi\sqrt{2}} \rho_f \sqrt{1 - 4\mu_f} \left[ 1 - 4\mu_f + (1 - 4|Q_f|s^2\kappa_f)^2 (1 + 2\mu_f) \right], \quad (15)$$

where

$$\rho_f = \frac{1 - \Delta r^{MSSM}}{1 - \hat{\Sigma}'_Z(M_Z^2)} + \frac{2}{a_f} \delta a_f, \quad (16)$$

$$\kappa_f = 1 - \frac{1}{4|Q_f|s^2a_f} \left( \delta v_f - \frac{v_f}{a_f} \delta a_f \right), \quad (17)$$

and

$$s^2 = \frac{1}{2} \left\{ 1 - \left[ 1 - \frac{A}{(1 - \Delta r^{MSSM})} \right]^{1/2} \right\} \quad (A \equiv \frac{4\pi\alpha}{\sqrt{2}G_F M_Z^2}). \quad (18)$$

Here  $N_c^f = 1$  (for leptons), 3 (for quarks),  $\mu_f \equiv m_f^2/M_Z^2$ ;  $v_f = (T_3^f - 2Q^f s^2)/2sc$  and  $a_f = T_3^f/2sc$  are the vector and axial coefficients of the neutral current [9]. The one-loop corrections to these coefficients are defined through the radiative shifts

$$v_f \rightarrow v_f + \delta v_f, \quad a_f \rightarrow a_f + \delta a_f. \quad (19)$$

These shifts, together with  $\Delta r$ , are to be computed in the MSSM and isolated their departure from the total “background” contribution of the RSM (see eqs.(6) and (11)). Similarly for the  $Z$  wave-function renormalization effects, which are represented in eq.(16) by the derivative of the corresponding renormalized self-energy :

$$\hat{\Sigma}'_Z(M_Z^2) = \Sigma'_Z(M_Z^2) - \frac{\Sigma^\gamma(k^2)}{k^2} \Big|_{k^2=0} + 2\frac{c^2 - s^2}{s c} \frac{\Sigma^{\gamma Z}(0)}{M_Z^2} - \frac{c^2 - s^2}{s^2} \left( \frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2} \right). \quad (20)$$

As for the renormalized  $\gamma Z$  mixed self-energy on the mass shell of the  $Z$ , it is included as a part of the total radiative shift of the vector coefficient as follows:

$$\delta v_f^{\gamma Z} = Q_f \frac{\hat{\Sigma}^{\gamma Z}(M_Z^2)}{M_Z^2} = Q_f \left\{ \frac{\Sigma^{\gamma Z}(M_Z^2) + \Sigma^{\gamma Z}(0)}{M_Z^2} - \frac{c}{s} \left( \frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2} \right) \right\}. \quad (21)$$

We remark that the total additional MSSM contribution from  $\Sigma^{\gamma Z}(k^2)$  with respect to the RSM turns out to vanish at  $k^2 = 0$ , and so  $\Sigma^{\gamma Z}(0)$  may actually be dropped from eqs.(12),

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<sup>6</sup>Although the (low-energy) renormalization framework of Ref.[27] is different from the one considered here, the unrenormalized expressions are the same and they have been straightforwardly adapted (as in Ref.[5]) to the counterterm structure of the minimal  $\alpha$ -scheme of Ref.[23] without changing the sign convention for the self-energy functions.

(20) and (21). The list of the Feynman diagrams and analytical MSSM contributions from vertices and self-energies to the form factors  $\rho_f$  and  $\kappa_f$  is provided in Ref.[22] and, as asserted, we shall not dwell here into their detailed structure. Expanding up to one-loop level the previous formulae, the general form of the SUSY correction to any given partial width can be written (in a notation partly inspired from Ref.[15])

$$\delta\Gamma^{SUSY}(Z \rightarrow f\bar{f}) = \Gamma_Z^0(G_F) \left[ \nabla_U^{SUSY} + \nabla_{Q_f}^{SUSY} + \nabla_{V_f}^{SUSY} \right], \quad (22)$$

where  $\Gamma_Z^0(G_F)$  is the tree-level width in the  $G_F$ -parametrization (defined by eq.(15) with  $\rho_f = \kappa_f = 1$ ), and

$$\begin{aligned} \nabla_U^{SUSY} &= \hat{\Sigma}'_Z(M_Z^2) - \Delta r^{SUSY} = \Delta\rho^{SUSY} + \dots \\ \nabla_{Q_f}^{SUSY} &= 2|Q_f| \frac{v_f a_f}{v_f^2 + a_f^2} \left( 4sc \frac{\hat{\Sigma}^{\gamma Z}(M_Z^2)}{M_Z^2} - \frac{A}{\sqrt{1-A}} \Delta r^{SUSY} \right) \\ &= 2|Q_f| \frac{v_f a_f}{v_f^2 + a_f^2} \left[ \left( \frac{A}{\sqrt{1-A}} - 4s^2 \right) \left( \frac{c^2}{s^2} \Delta\rho^{SUSY} \right) + \dots \right], \\ \nabla_{V_f}^{SUSY} &= \delta\rho_{V_f} - 8|Q_f| s^2 \frac{v_f a_f}{v_f^2 + a_f^2} \delta\kappa_{V_f} = 2 \left( \frac{v_f \delta v_f + a_f \delta a_f}{v_f^2 + a_f^2} \right)_{vertices}, \end{aligned} \quad (23)$$

$\nabla_U$  being a universal correction,  $\nabla_{Q_f}$  applies only to charged fermions and  $\nabla_{V_f}$  is the vertex correction. For the first two types of corrections, we have singled out the  $\Delta\rho^{SUSY}$  component entering the full expressions, though this does not mean that the rest of the contributions are comparatively negligible, in contrast to what happens in the SM case. As for the structure of  $\nabla_{V_f}$ , it is too complicated to be discussed here in any detail [22].

It should be emphasized that in the MSSM all potential quantum effects that entail a departure with respect to the RSM are subdued by the decoupling theorem. This is because the breaking of SUSY is independent of the breaking of the gauge symmetry. Therefore, in view of the current limits on sparticle masses ( $m_{SUSY} \geq \mathcal{O}(M_Z)$ ), the SUSY quantum effects are generally expected to be tiny as compared to quantum effects from the RSM. This is indeed the case for the radiative corrections to  $M_W$  [2, 5, 6], the reason being that, in contradistinction to the light fermions in the SM, no leading RG-corrections (first term on the RHS of eq.(14)) from sparticles are possible in the MSSM. For the  $Z$ -width, however, this feature has a lesser impact, since  $\Delta\alpha$  cancels out in  $\rho_f$  [9] (see later on). We are thus left with oblique corrections at the next-to-leading order (second and third terms on the RHS of eq.(14)), plus non-oblique contributions driven by significant Yukawa couplings. In both cases, the SM and SUSY effects may be of the same order of magnitude, if sparticles are not too heavy. Among the next-to-leading oblique corrections, we have the custodial symmetry breaking ones. However, custodial symmetry cannot be broken in the MSSM by non-decoupling SUSY effects, whether statical ( $\Delta\rho$ )

or dynamical (wave-function renormalization of the gauge bosons). For large enough  $m_{SUSY}$ , oblique and non-oblique SUSY corrections must go to zero. We have checked this analytically in our calculations, and also numerically in our computer codes. For example, consider the SUSY mass parameters  $(\mu, M)$  associated to the higgsino-gaugino parameter space [1]. In the mass-eigenstate basis, the 2-component gaugino and higgsino Weyl spinors combine among themselves to form 4-component charginos and neutralinos (“inos”) and the corresponding neutral current is in general an admixture of vector and axial components (as is also the case for the neutral current associated to conventional fermions). However, for  $M, \mu \rightarrow \infty$  the SUSY neutral current becomes a pure vector-like current and a standard vector Ward identity insures an exact cancellation of the renormalized vertex functions constructed from “inos” and sfermions in that limit. In short: we expect to see measurable SUSY quantum effects on the  $Z$ -width only if the soft SUSY-breaking mass parameters are not much larger than the electroweak scale. In such circumstances, specific corrections (e.g. electroweak) from SUSY can be comparable or even larger than in the SM, as it happens to be the case e.g. for the electroweak corrections to the top quark width [14] and also in the present case for the  $Z$ -width. This will be demonstrated below.

We come now to explicit numerical results. The actual calculation to obtain these results is rather cumbersome, since we retain exact dependence on all masses and keep track of all matrix coupling constants for all SUSY particles in their respective mass-eigenstate bases [22]. The computation of the many 3-point functions involved has been carried out using thoroughly tested standard techniques based on exact reduction formulae and subsequent expansion of the scalar functions in terms of complex Spence functions [28]. In Figs.1a and 1b we display contour lines of constant value of the genuine SUSY correction defined in eqs.(6), (22)

$$\delta\Gamma_Z^{SUSY} = \sum_{l=e,\mu,\tau} [\delta\Gamma^{SUSY}(Z \rightarrow l^+ l^-) + \delta\Gamma^{SUSY}(Z \rightarrow \nu_l \bar{\nu}_l)] + \sum_{q=u,d,c,s,b} \delta\Gamma^{SUSY}(Z \rightarrow q\bar{q}) \quad (24)$$

in a standard window of the higgsino-gaugino  $(\mu, M)$ -space for Models I and II, respectively. In both models, the pattern of sfermion masses is generated from the generic formula [1]

$$m_{\tilde{f}_{L,R}}^2 = m_f^2 + M_{\tilde{f}_{L,R}}^2 \pm \cos 2\beta (T_{L,R}^3 - Q_{\tilde{f}} s^2) M_Z^2, \quad (25)$$

where the same soft SUSY-breaking parameter  $M_{\tilde{f}_L}^2$  is shared by the two members of any  $SU(2)_L$  doublet. However, while in Model I the R-type sfermions are assumed to be degenerate in mass with the L-type ones, this is not so in Model II where one further specifies the structure of the soft SUSY-breaking mass parameters in the standard manner suggested by radiative symmetry-breaking models (such as supergravity inspired

models [1]), namely:  $M_{\tilde{f}_{L,R}}^2 = m_0^2 + C(\tilde{f}_{L,R}) M^2$ , where  $m_0$  is a universal soft SUSY-breaking scalar mass at the GUT scale and  $C(\tilde{f}_{L,R})$  are certain RG-driven coefficients [29]. Altogether these models cover a sufficiently wide range of phenomenologically allowed masses for sneutrinos ( $\tilde{\nu}_l$ ), charged sleptons and squarks ( $\tilde{l}_a^\pm, \tilde{q}_a$  ( $a = 1, 2$ )), charginos ( $\Psi_i^\pm, i = 1, 2$ ) and neutralinos ( $\Psi_\alpha^0, \alpha = 1, \dots, 4$ ), whose present lower limits are [30]

$$m_{\tilde{l}_a^\pm} \geq 45 \text{ GeV}, \quad m_{\tilde{\nu}_l} \geq 42 \text{ GeV}, \quad m_{\tilde{q}_a} \geq 130 \text{ GeV}, \quad (26)$$

$$M_{\Psi_i^\pm} \geq 47 \text{ GeV}, \quad M_{\Psi_\alpha^0} \geq 20 \text{ GeV}. \quad (27)$$

It should be pointed out that the squark mass limits from Tevatron are not fully model independent [30] and in particular they depend on assumptions on the gluino masses and on canonical SUSY decay modes. As a consequence, a stop squark  $\tilde{t}_1$  could still be rather light ( $\lesssim M_Z/2$ ), a feature that can be easily accommodated in model building through a large mixing term  $m_t M_{LR}$  (proportional to the top quark mass  $m_t$ ) in the stop mass matrix. In spite of the fact that we have not included this term for the third squark family in eq.(25), we shall amply exploit this possibility in Part II. Here we prefer to present more conservative results by keeping all squark generations alike, i.e. without mixing. As in Ref. [5], we have furthermore imposed on our numerical analysis the condition

$$|\Delta\rho|^{SUSY} < 0.005. \quad (28)$$

In Fig.1 we have fixed the sfermion spectrum (25) with  $\tan\beta = 8$ ,  $m_{\tilde{\nu}_l} = 50 \text{ GeV}$  and  $m_{\tilde{u}} = m_{\tilde{c}} = m_{\tilde{b}} = 130 \text{ GeV}$ , i.e. a spectrum perfectly consistently with the bounds (26). In particular, our conservative choice for  $m_{\tilde{b}}$  implies, via eq.(25) with  $m_f = m_t$ , rather heavy ( $\simeq 200 \text{ GeV}$ ) partners of the top quark. It becomes patent from Fig.1a that genuine SUSY contributions of order  $\delta\Gamma_Z^{SUSY} \gtrsim +10 \text{ MeV}$  can comfortably be achieved in Model I, for a wide range of chargino masses. The fact that light charginos, i.e. charginos corresponding to points  $(M, \mu)$  near the boundary of the phenomenologically allowed region in Fig.1a, are responsible for a minimum SUSY correction to the  $Z$ -width is related to the large (oblique) negative contributions from the “ino” sector near that boundary. (We shall take advantage of this feature in Part II in connection to the analysis of  $R_b$  in the MSSM). However, in the middle region ( $M \simeq |\mu| \gtrsim 100 - 150 \text{ GeV}$ ), the latter contributions are positive (though smaller in absolute value) and add up to the oblique sfermion contributions plus the non-oblique (vertex) corrections, which are also globally positive in this case, and consequently  $\delta\Gamma_Z^{SUSY}$  increases up to a maximum  $\simeq +13 \text{ MeV}$ . Of course, away from the local maximum, the total SUSY contribution diminishes for larger and larger values of  $(\mu, M)$ , but it does not go to zero; instead,  $\delta\Gamma_Z^{SUSY}$  tends asymptotically to the constant positive effect from the sfermion self-energies, which have

fixed values for their masses, and it would only decouple upon simultaneous increase of the latters. It is worthwhile to note that sufficiently away from the neighbourhood of the boundary (specifically, for  $M, |\mu| > 150 \text{ GeV}$ ) the SUSY quantum effects described in Fig.1a originate from an average SUSY spectrum which is beyond the capability of pair production at LEP 200. Indeed, given the values of the sfermion masses fixed above, in this region we have

$$m_{\tilde{l}_a^\pm} \geq 94 \text{ GeV}, \quad m_{\tilde{q}_a} \geq 130 \text{ GeV}, \quad M_{\Psi_i^\pm} \geq 94 \text{ GeV}, \quad M_{\Psi_\alpha^0} \geq 55 \text{ GeV}. \quad (29)$$

Remarkably enough, it turns out that even in this LEP 200 unaccessible region, the quantum correction  $\delta\Gamma_Z^{SUSY}$  may well reach the level of the total error (theoretical plus experimental, added in quadrature) in eqs.(2,3-4) and it therefore leaves open the possibility to potentially detect these extra effects in the future. It follows that in most of the window of Fig.1a, the average electroweak SUSY correction to the  $Z$ -width is numerically larger, but opposite in sign, as compared to the (negative) electroweak SM correction  $\delta\Gamma_Z^{(ew)SM} = \Gamma_Z^{(ew)SM} - \Gamma_Z^0(G_F)$  with respect to the SM tree-level width in the  $G_F$ -parametrization. We have checked this explicitly using the upgraded version of the computer code BHM [31], with the inputs:  $M_Z$  from eq.(1),  $m_t = 174 \text{ GeV}$ ,  $M_H = 100 \text{ GeV}$ <sup>7</sup> and then projecting the electroweak part, with the result  $\delta\Gamma_Z^{(ew)SM} = -9.5 \text{ MeV}$ . Therefore, if the physical width of the  $Z$  turns out to be the one predicted by the MSSM, eq.(5), the total electroweak correction with respect to the tree-level width  $\Gamma_Z^0(G_F)$  in the SM (now the RSM) will be (neglecting for the moment the extra Higgs correction)

$$\delta\Gamma_Z^{(ew)MSSM} = \Gamma_Z^{(ew)MSSM} - \Gamma_Z^0(G_F) = \delta\Gamma_Z^{(ew)SM} + \delta\Gamma_Z^{SUSY} \simeq +(1-3) \text{ MeV}. \quad (30)$$

This will hold true for a SUSY spectrum satisfying eq.(29). For demonstration purposes we have tolerated an overall SUSY correction slightly exceeding the total theoretical plus experimental error from eqs.(2,3-4), just to exhibite the potentiality of the supersymmetric virtual effects from a sparticle spectrum amply complying with the present phenomenological bounds. For heavier and heavier sparticles, the correction decreases and can be made numerically very close to the electroweak correction, but opposite in sign. The resulting cancellation could effectively render invisible the total electroweak correction in the MSSM, even assuming a substantial improvement of the theoretical errors. We conclude that the effect might perhaps be discovered either by “missing” the expected electroweak correction in the SM, or even finding that it goes in the opposite direction. Of course, as we have advertised, this conclusion would only apply if the negative Higgs effects in the MSSM ( $\delta\Gamma_Z^H$  in eq.(6)) are kept very small (in absolute value) with respect

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<sup>7</sup>Since we are assimilating the SM to the RSM, the lightest neutral Higgs mass must be less than about  $130 \text{ GeV}$  [21].

to  $\delta\Gamma_Z^{SUSY}$ , or if we confine our study into a region where the additional Higgs effects from the MSSM are always positive. Indeed, as shown in Part II, we are naturally led to a particular scenario like this when trying to cope with the “ $R_b$  crisis” in the MSSM. Notwithstanding, it should be mentioned that global negative corrections  $\delta\Gamma_Z^{MSSM}$  are still possible by picking contrived values for the parameters.

The corresponding contour lines for Model II are shown in Fig.1b. In this case the sfermion masses are controlled by the single parameter  $m_0$ . Since a minimum value common for squarks and sleptons must be found out for this parameter ( $m_0 \gtrsim 60\text{ GeV}$ ) in order to simultaneously fulfil all the phenomenological bounds (26),(27), the average sfermion spectrum in Model II turns out to be far more heavier than in Model I. For example, in the middle region of Fig.1b ( $M, |\mu| \geq 150\text{ GeV}$ ) the sfermion spectrum in Model II satisfies

$$m_{\tilde{l}^\pm}, m_{\tilde{\nu}_l} > 100\text{ GeV}, \quad m_{\tilde{q}} > 400\text{ GeV}. \quad (31)$$

Therefore, the corrections are generally smaller (about a factor of  $2 - 3$ ) than in Model I, though still not fully negligible: around the maximum,  $\delta\Gamma_Z^{SUSY} \simeq +6\text{ MeV}$ , i.e. numerically close to the SM electroweak correction [9, 31], and reversed in sign. Notice that there are no threshold effects neither in Model I nor in Model II associated to the wave function renormalization of the  $Z$  gauge boson, eq.(20). In fact, while sfermion and chargino masses are always assumed to be heavier than half the  $Z$ -mass, the neutralino masses may continuously approach points of  $(\mu, M)$ -space where  $M_\alpha^0 + M_\beta^0 = M_Z$ . However, the corresponding singularity in  $\hat{\Sigma}'_Z(M_Z^2)$  cancels out identically and no threshold effect remains.

For any channel  $Z \rightarrow f\bar{f}$ ,  $\nabla_{U,Q}^{SUSY}$  from eq.(23) give the leading contribution, which is numerically very close for the two  $T^3 = \pm 1/2$  components in each fermion doublet. Therefore, the vertex correction,  $\nabla_{V_f}^{SUSY}$ , which is negative for the up components and positive for the down components, gives the bulk of the difference between the corrections to the  $Z$  partial widths into  $T^3 = +1/2$  fermions and  $T^3 = -1/2$  fermions (cf. Figs.2-5). It is worth mentioning that although  $\nabla_{U,Q}^{SUSY}$  are leading, there are strong cancellations between the two terms in these formulas; for example, as seen on the first eq.(23), the  $\Delta\alpha$  contribution from eq.(14) exactly cancels in the difference  $\nabla_U^{SUSY}$ , leaving  $\Delta\rho^{SUSY}$  as one of the leading remainders. On the other hand, the vertex correction is specially significant for the  $b\bar{b}$  channel, where the Yukawa couplings can be substantially large. Nevertheless, in contrast to the SM, where there is an overcompensation of the (positive) propagator correction by the (negative) vertex correction [9], it turns out that the extra non-oblique SUSY contribution is of the same order of magnitude and has the same (positive) sign as the extra oblique contribution from  $\Delta\rho^{SUSY}$ . As a consequence, the two effects turn out to add up and might give rise to a measurable correction. We show this explicitly in

Figs.2a-2d, where we fix  $(\mu, M) = (-100, 100) \text{ GeV}$  and plot the dependence of the partial width corrections,  $\delta\Gamma_Z^{SUSY}(Z \rightarrow f\bar{f})$ , on the squark and slepton masses in Model I. Upon inspection of these figures we gather three noticeable facts: i) the SUSY corrections to the partial widths are about  $4 - 8$  times larger for quarks than for leptons, ii) the corrections to the partial widths into  $T^3 = -1/2$  fermions are larger than the corrections to the partial widths into  $T^3 = +1/2$  fermions, the  $b\bar{b}$  channel being the most favoured one, and iii) there is a moderate decrease of the corrections for heavier and heavier sfermions masses; e.g. in Fig.2a,  $\delta\Gamma^{SUSY}(Z \rightarrow b\bar{b})$  roughly decreases from  $4 \text{ MeV}$  to  $2.7 \text{ MeV}$  when  $m_{\tilde{u}}$  increases from  $100 \text{ GeV}$  to  $200 \text{ GeV}$ . The decrease is even slower in Fig.2c, where the partial widths into quarks are confronted with increasing slepton masses, which enter through oblique corrections the weak gauge boson propagators. Similar considerations apply, respectively, to Figs.2b and 2d. The corresponding plots for Model II are shown in Figs.3a-3b. In this case the relevant parameter is the universal soft SUSY-breaking sfermion mass  $m_0$ . Notice that some of the corrections may be slightly negative, as is the case for the neutrino channels  $\nu_l\bar{\nu}_l$  in Fig.3b.

In Figs.4a-4b, we study the evolution of the radiative corrections on the parameter  $\tan\beta$ , only for Model I, where the effects are bigger. The partial widths into leptons and quarks are separately considered. There we see why we have chosen  $\tan\beta = 8$  in Fig.1; it corresponds to a an approximate saturation value for the leading ( $q\bar{q}$ ) contributions in the range  $\tan\beta \lesssim 50$ . For the leptons, however, especially for the  $\tau^+\tau^-$  and  $\nu_\tau\bar{\nu}_\tau$  channels, there is still an ongoing evolution beyond  $\tan\beta \geq 8$ , though the relative impact on  $\delta\Gamma_Z^{SUSY}$  is minor. Remember that  $\tan\beta$  enters the mass matrices for chargino-neutralinos and also determines potentially relevant fermion-sfermion-chargino/neutralino Yukawa couplings in the vertex corrections  $\nabla_{V_f}^{SUSY}$  on eq.(23). The most significant ones are

$$h_t = \frac{g m_t}{\sqrt{2} M_W \sin\beta} \quad , \quad h_b = \frac{g m_b}{\sqrt{2} M_W \cos\beta} , \quad (32)$$

which contribute to the SUSY effects on the  $Zb\bar{b}$  vertex, and

$$h_\tau = \frac{g m_\tau}{\sqrt{2} M_W \cos\beta} , \quad (33)$$

which goes into the structure of the  $Z\tau^+\tau^-$  and  $Z\nu_\tau\bar{\nu}_\tau$  vertices. The Yukawa coupling associated to the charm quark,  $h_c$ , is not sizeable enough since  $m_c/M_W$  is too small and  $h_c$  does not increase with  $\tan\beta$ . That is why the contributions to the partial widths into the modes  $u\bar{u}$  and  $c\bar{c}$  are practically indistinguishable in our figures. Obviously, another point where to focus our attention is on the dependence of  $\delta\Gamma_Z^{SUSY}$  on the top quark mass (Figs.5a-5b). Since this parameter is involved in the sbottom-stop mass differences, all partial widths are universally affected (through propagator corrections) by the precise

value of  $m_t$ , and  $\delta\Gamma^{SUSY}(Z \rightarrow b\bar{b})$  is additionally affected by important vertex corrections associated to an enhancement of  $h_t$  for increasing  $m_t$ . Indeed, the slope of the  $b\bar{b}$  curve can be seen from Fig.5a to be more pronounced than for the other channels. As a matter of fact, of the three Yukawa couplings (32)-(33), the most significant one is  $h_t$ . It basically determines the important non-oblique SUSY corrections responsible for the gap between the  $b\bar{b}$  channel with respect to the the  $s\bar{s}$  and  $d\bar{d}$  modes in Figs.2-5. For large enough  $\tan\beta$ , the corrections to the  $b\bar{b}$  channel are further enhanced by an increasing  $h_b$  (with  $m_b = 5\text{ GeV}$ ), as can be appreciated in Fig.4a. To be more specific, there is a balance between chargino and neutralino contributions in the vertex corrections; namely, the chargino vertex corrections diminish with  $\tan\beta$  whereas the neutralino vertex corrections increase with it. Only for large enough  $\tan\beta$  the neutralino effects are overwhelming; for example, for  $\tan\beta \gtrsim 70$  (where we are bordering the limits of validity of perturbation theory) and keeping the same values for the rest of the parameters,  $\delta\Gamma^{SUSY}(Z \rightarrow b\bar{b}) \gtrsim 4\text{ MeV}$ . On the other hand, even though we wish not to emphasize the region of  $\tan\beta < 1$ , it is worthwhile to note that, there,  $h_t$  grows so fast that one could easily obtain similar staggering effects as before, viz.  $\delta\Gamma^{SUSY}(Z \rightarrow b\bar{b}) \gtrsim 4\text{ MeV}$  for  $\tan\beta \lesssim 0.7$ <sup>8</sup>. In this region, which we have not highlighted in Fig.4, there is in fact an unbounded, positive, contribution at the left edge of the curves. Nevertheless we shall see in Part II that this contribution is compensated by an overwhelming negative effect from  $\delta\Gamma_Z^H$ . We remark that the point  $\tan\beta = 1$  is the site of the absolute minimum for each of the partial widths, since at this point the sfermion generations have minimal mass splittings (cf.eq.(25)) and hence  $\Delta\rho^{SUSY}$  takes on its smallest value.

In conclusion, there could be significant, genuinely supersymmetric, electroweak renormalization effects on the  $Z$ -width in the MSSM. We have explored a sufficiently wide domain of the SUSY parameter space in support of this fact. In particular, we have concentrated our numerical analysis on regions corresponding to rather conservative values for all sparticle masses; thus, although it is not strictly required by the phenomenological bounds, we have assumed masses of  $\mathcal{O}(100)\text{ GeV}$  for the superpartners of the top quark, and in general we have put more emphasis on the results obtained for a full SUSY spectrum above the possibilities of pair production at LEP 200. Even with these hypotheses, the size of the ensuing SUSY corrections could easily reach the level of the present theoretical errors. For the recent CDF values of the top quark mass, we have found natural windows of parameter space where there could be a large cancellation, or even an overcompensation, of the expected electroweak SM corrections by the electroweak supersymmetric quantum effects. Upon relaxing the previous hypotheses on sparticle masses

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<sup>8</sup>Scenarios with  $\tan\beta < 1$  are generally disfavoured [32] but still some reduced interval  $0.7 \lesssim \tan\beta < 1$  is in dispute.

to the strict limit placed by the present LEP 100 bounds, more freedom is obtained to test the potentiality of the SUSY corrections in  $Z$ -decay physics. This point will be re-taken and fully exploited in Part II. We hope that in the future a better determination of  $m_t$ ,  $\alpha_s$  and the parameters of the Higgs sector, will allow to uncover the electroweak quantum effects on  $\Gamma_Z$ . Missing of these effects, or finding them opposite in sign to SM expectations, could be interpreted as indirect evidence of SUSY.

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## Figure Captions

- **Fig.1** (a) Contour plots in the higgsino-gaugino  $(\mu, M)$ -parameter space for the total SUSY correction  $\delta\Gamma_Z^{SUSY}$  in Model I. The sfermion spectrum is obtained from eq.(25) with  $\tan\beta = 8$ ,  $m_{\tilde{\nu}_l} = 50 \text{ GeV}$  and  $m_{\tilde{u}} = m_{\tilde{e}} = m_{\tilde{b}} = 130 \text{ GeV}$ . The top quark mass is fixed at  $m_t = 174 \text{ GeV}$ . The blank regions are phenomenologically excluded by the constraints  $M_{\Psi_i^\pm} > 47 \text{ GeV}$ ,  $M_{\Psi_\alpha^0} > 20 \text{ GeV}$ ; (b) As in case (a), but for Model II and fixed  $m_0 = 63 \text{ GeV}$ .
- **Fig.2** SUSY corrections to the various partial widths  $\Gamma(Z \rightarrow f\bar{f})$  in Model I as a function of the squark masses (cases (a) and (b)) and slepton masses (cases (c) and (d)). We have taken  $(\mu, M) = (-100, 100) \text{ GeV}$  and the other non-varying parameters as in Fig. 1a.
- **Fig.3** As in Fig.2, but for Model II as a function of  $m_0$ .
- **Fig.4** Dependence of  $\delta\Gamma^{SUSY}(Z \rightarrow f\bar{f})$  on  $\tan\beta$  for  $Z$  decaying into (a) quark-antiquark, and (b) lepton-antilepton in Model I. The remaining fixed parameters are chosen as in Fig.1a and Fig.2.
- **Fig.5** Dependence of  $\delta\Gamma^{SUSY}(Z \rightarrow f\bar{f})$  on  $m_t$  within the CDF mass limits, for  $Z$  decaying into (a) quark-antiquark, and (b) lepton-antilepton in Model I. Remaining parameters as in Fig.1a and Fig.2.

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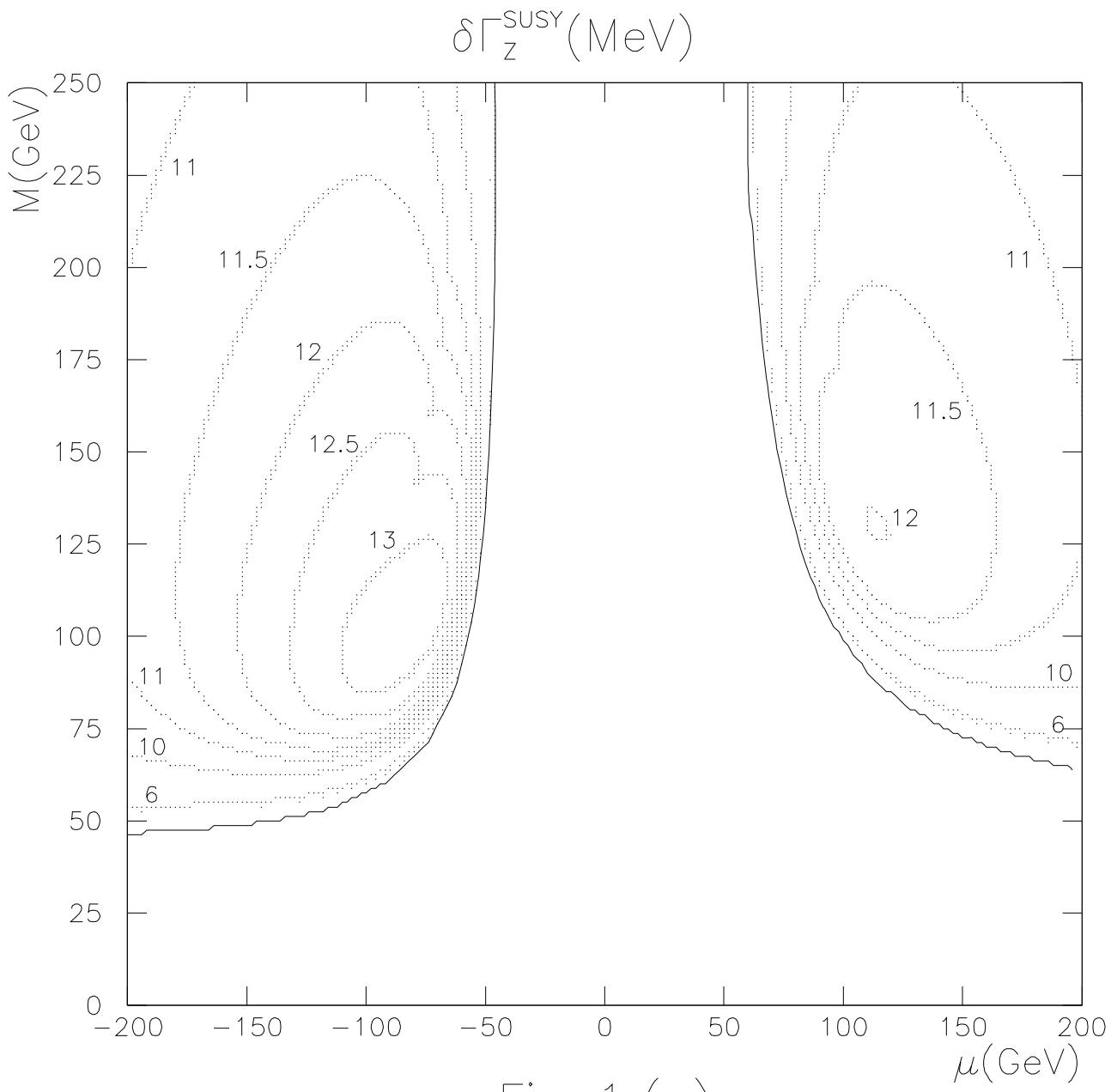


Fig. 1 (a)

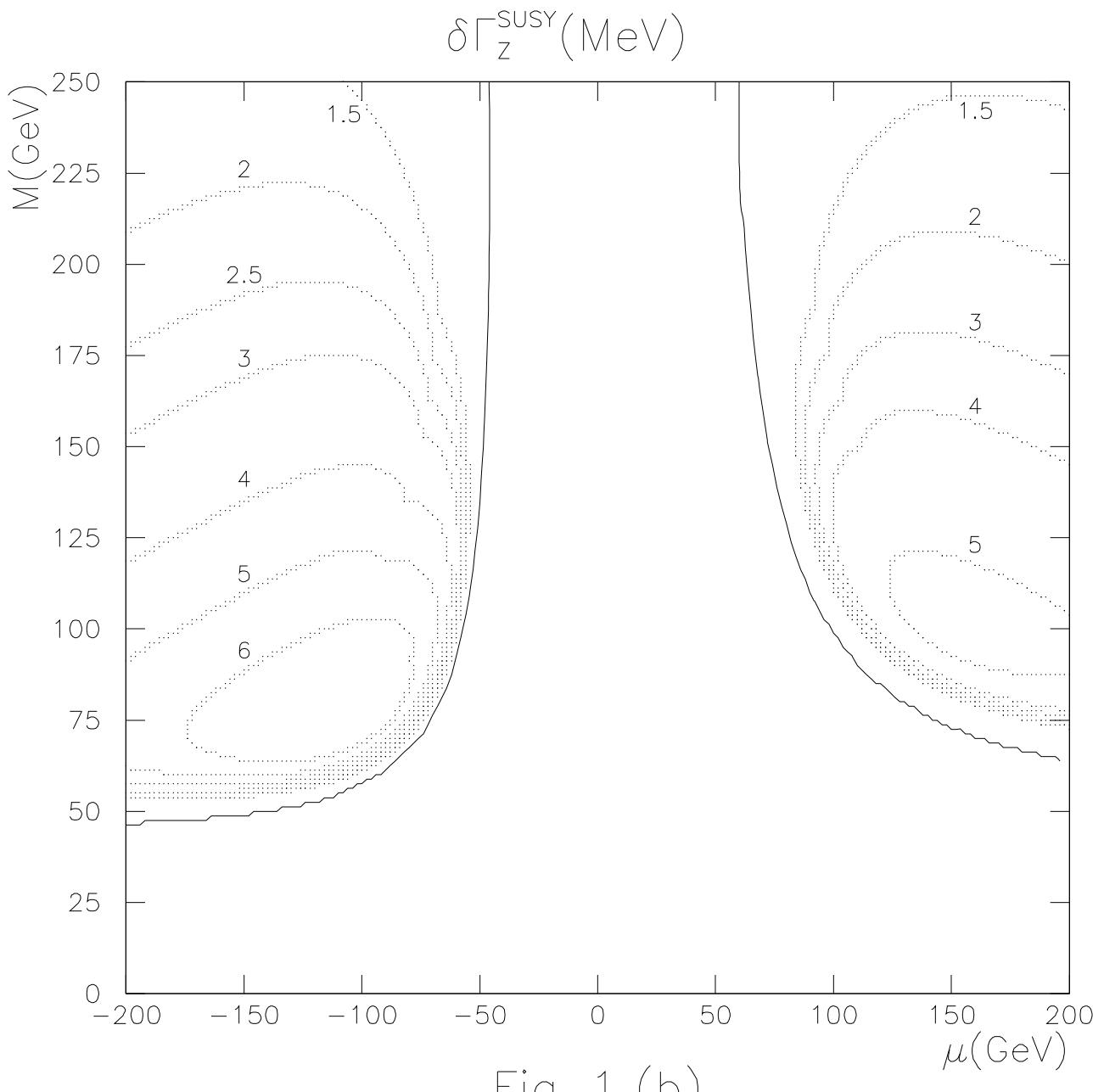


Fig. 1 (b)

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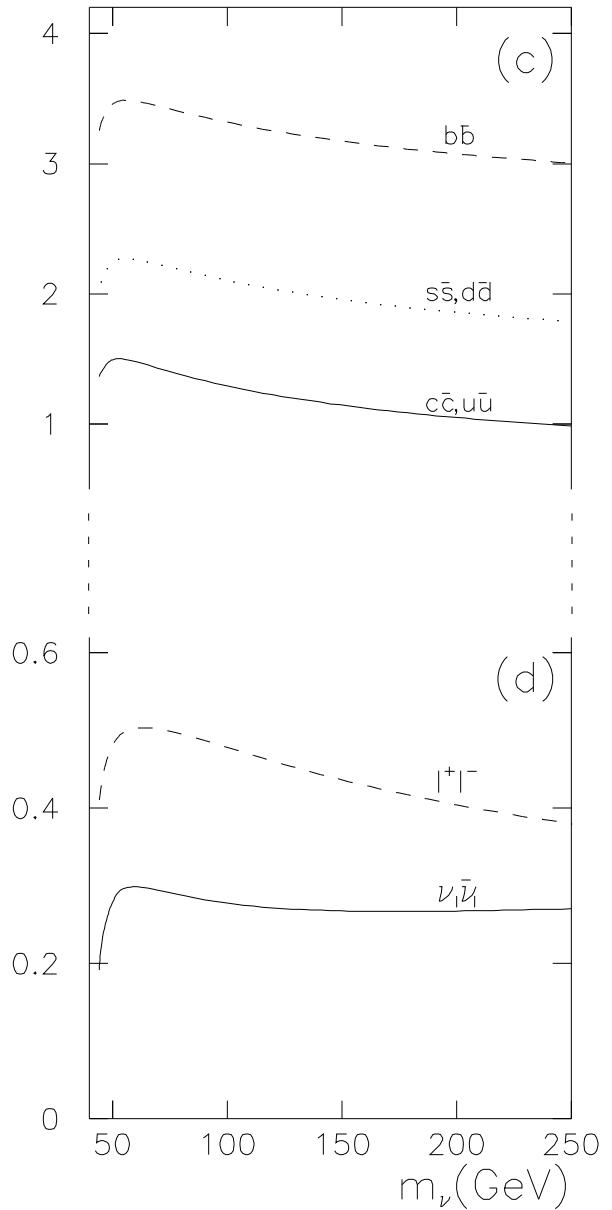
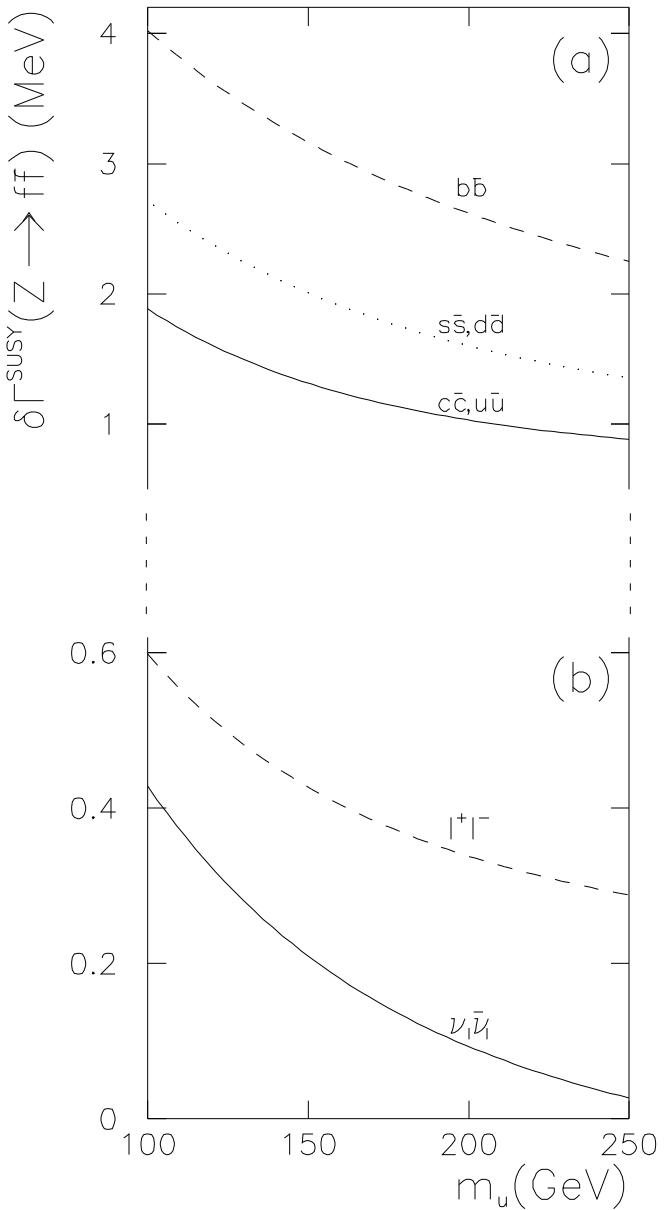


Fig. 2

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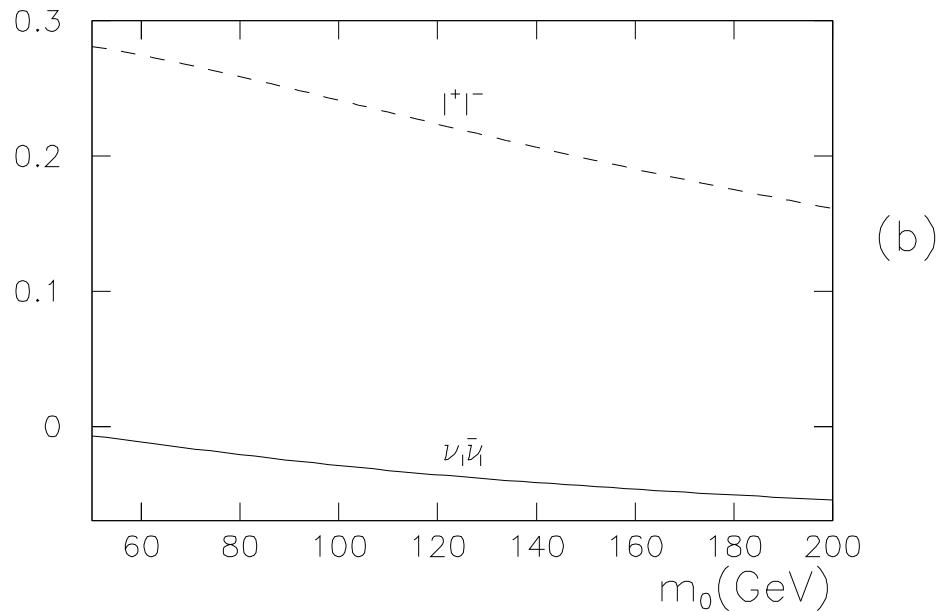
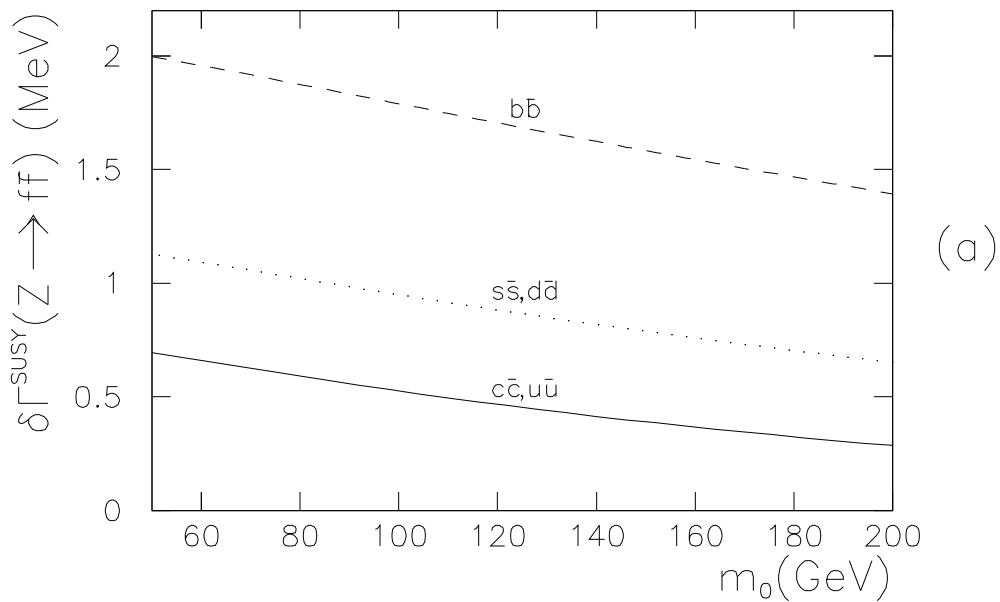


Fig. 3

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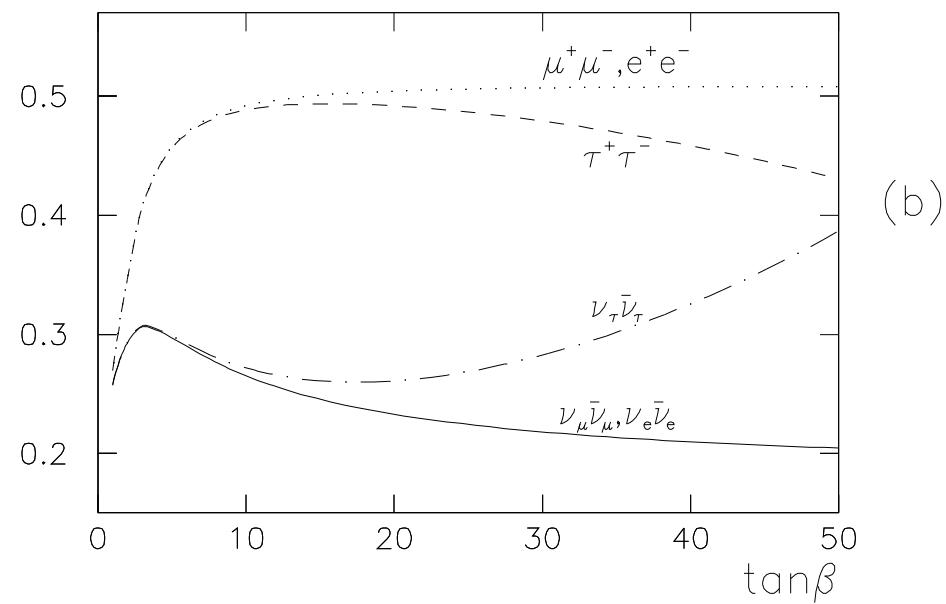
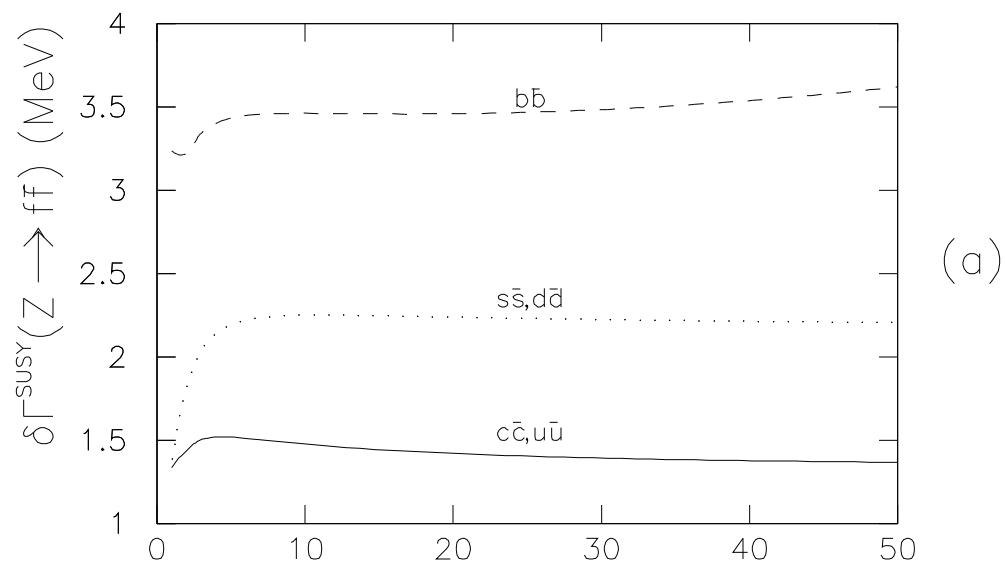


Fig. 4

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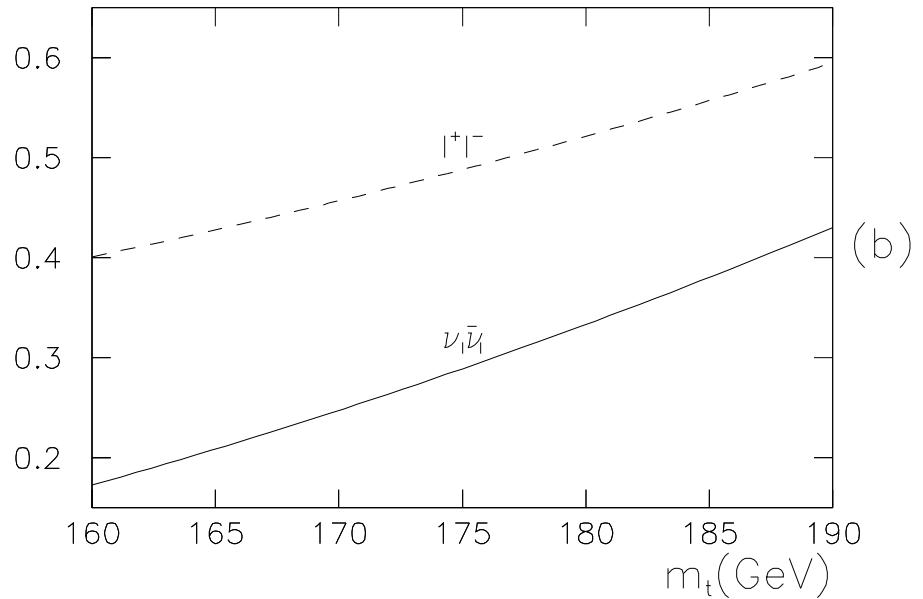
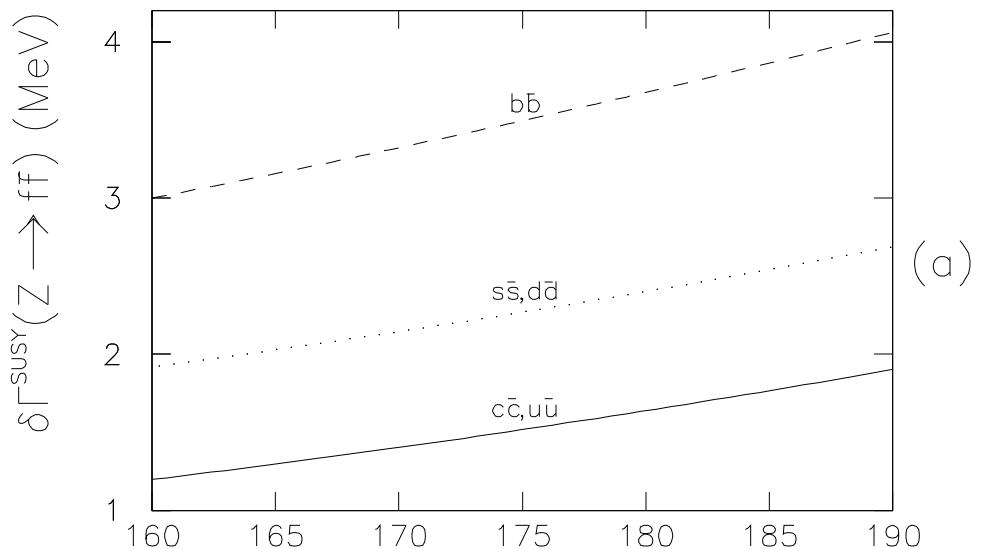


Fig. 5

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